

**THE STRUCTURE OF CULM FORM OF *Phyllostachys pubescens***

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**Abstract**

Based on 3567 culms from 13 districts located in China and Japan, fourteen mathematical models have been worked out by statistical procedures. They are

**1. The distribution of internode length of culms**

$$h = \left( \frac{H}{1.25N} \right) \text{Exp} - 2 \left[ \frac{N}{N} - 1 \right]^2$$

$$H_{(p)} = \frac{H}{1.25N} \int_1^N \text{Exp} - 2 \left[ \frac{N}{N} - 1 \right]^2 dN$$

**2. The distribution of internode circumference of culms:**

$$L = A \left( 1 - \frac{N}{2N} \right)$$

**3. The distribution of internode wall thickness of culms**

$$Z = B \left( 1 - \frac{N}{2N} \right)$$

**4. The distribution of internode volume of culms**

$$V = \frac{H}{1.25N} \left( 1 - \frac{N}{2N} \right)^2 \left( A - \pi \frac{B}{2} \right)^2 \text{Exp} - 2 \left[ \left( \frac{N}{N} \right) - 1 \right]^2$$

$$V = \frac{H}{1.25N} \left( A - \pi \frac{B}{2} \right)^2 \int_1^N \left( 1 - \frac{N}{2N} \right)^2$$

$$\text{Exp} - 2 \left\{ \left( \frac{N}{N} - 1 \right) \right\} dN$$

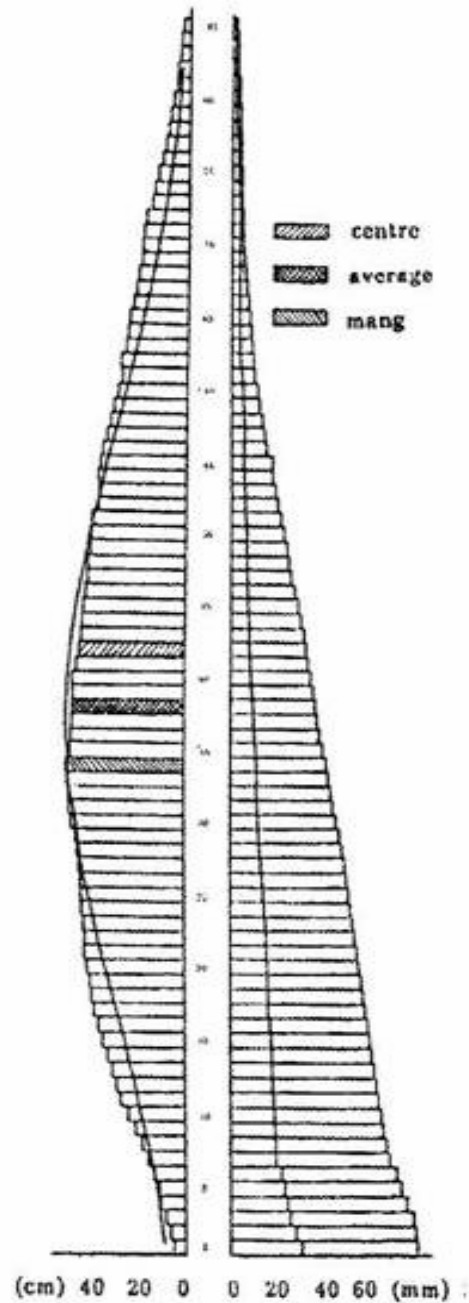


Fig. 5-1, Internodes length(a),circum ference(b)and wall thickness(c) of culms(Yixing,Jingsu,1980)

**5. The structural model of the relative diameter of culms(D%)**

$$D\% = 122 [1 - (0.1H_x)^3] - 18.5H_x (1 - 0.1H_x)$$

Fig.5-1 Internoda length(a),circum ference (b)and wall thickness (c) of culms (Yixing,Jiangsu,1980)

**6. The structural model of the relative wall thickness of culms(Z%)**

$$Z\% = 107.5-10.52H_x+0.296H_x^2-0.0118H_x^3$$

**7. The structural model of relative fresh weight of culms(W%)**

$$W\% = 27H_x-2.5H_x^2+0.08H_x^3$$

**8. The relationship between the culm breast-height diameter (D) and the culm length (H) which is dependent on the annual mean temperature (t) and rainfall (R):**

$$H = (0.1t+0.0216 \frac{R}{t} -1.1) D^{(10223-28.6 \frac{R}{t}) \cdot 10^{-4}}$$

**9. The relationship between the breast-height diameter (D) and the fresh weight (W) which also depends on the annual mean temperature (t),**

$$W = (163+20.8t+9.01 \frac{R}{t}) \cdot 10^{-4} \times D [253055-564.8t-187.4 \frac{R}{t}] \cdot 10^{-5}$$

**10. The relationship between the breast-height diameter (D) and the clear length**

(H<sub>a</sub>) which is dependent on the annual mean temperature (t) and rainfall (R) H<sub>a</sub>=

$$(0.667Rt^2 \times 10^{-6}+0.433)D-1$$

**11. The relationship between the breast-height diameter (D) and the total fresh weight of branches with leaves (W<sub>bl</sub>) which also result from the annual mean temperature (t) and rainfall (R)**

$$W_{bl} = [0.188 \times (R^2/t)10^{-5} +0.5] D-1.6$$

**12. The relationship between the diameter (D) at breast-height and the diameter (D<sub>x</sub>) at any height:**

$$D_x = (109.4-9.5H_x+0.14H_x^2-0.028H_x^3) D / (81.14+2.85D-0.085D^2)$$

**13. The relationship between the breast-height diameter (D) and the wall thickness (Z<sub>x</sub>) at any height**

$$Z_x = [(107.5-10.52H_x+0.296H_x^2-0.0118H_x^3) D^{(2.14D+81.44)} (0.72-1.85Rt10^{-5}+0.086D)]$$

**14. The relationship between the breast height diameter (D), the length (H) and the total fresh weight (W) in connection with the influence of annual mean temperature (t) and rainfall (R)**

$$W = [(2t + \frac{R}{t} + 8) / (100t + 20 \frac{R}{t} - 920)] HD^{(150825-564.8t+98.6 \frac{R}{t}) \cdot 10^{-5}}$$

Table 5-1 Natural conditions of sampled districts

Locations	N.Lat.	E.Long	An.mtemp.(°C)	An.ppt.(mm)	R/t	Plots	Culms
Chaoping	24°15'	110°45'	19.97°	2142	107.3	14	20
Zhungyi	25°50'	114°10'11	19.20°	1484	77.3	4	65
Huankeng	27°34'	7°40'105°0	17.75°	1880	105.9	1	45
Zhanning	28°19'28°	0'105°42'1	16.50°	1350	81.8	7	449
Chisui	35'	05°23'117°	18.10°	1350	73.0	5	73
Fuxing	28°42'	48'121°16'	17.40°	1844	106.0	6	109
Taimaoshan	28°45'	118°00'11	17.60°	1800	102.3	65	575
Shimen	29°37'	9°25'119°5	15.98°	1512	94.6	28	53
Yuchiashan	30°00'	1'119°00'1	15.60°	1615	103.5	6	54
Tienmoshan	30°25'	35°04'	14.40°	1800	125.0	11	82
Yixing	31°22'		15.60°	1320	84.6	92	1126
Xiashui	31°55'		15.80°	970	61.4	2	647
Kyoto	35°00'		16.31°	1537	94.2	-	289
Total						244	3567

**Node Distribution of Culms**

An attempt is made to analyse the structure of culm form of *Phyllostachys pubescens* including diameter, height, wall thickness and weight with special reference to the climatic conditions. *Phyllostachys pubescens* is the

most important timber bamboo in China and naturally distributed in the region between 23°~33°N and 104°~122°E with an elevation of less than 1,200 metres. In this region annual mean temperature varied from 12°C to 22 °C nad annual precipitation from 800mm to

2,200 mm. During the period of 1958~1980, we investigated 3,298 culms of 244 plots from 13 districts located in China and coupled with the data of 289 culms from Japanese literature as listed in Table 5-1.

Node Distribution of Culms Nodes of individual culms of *Phyllostachys pubescens* var greatly. Generally, a large culm may have 80~90 nodes, while a small one only have 50 nodes. The position of nodes is indicated by 1,2,3...N from the base to the top of a culm. The length, circumference and wallthickness of individual internodes can be diagrammed accordingly to show a general pattern of their distribution (Fig.5-1).

### 1. Distribution of internodal length

As seen from Fig.4-1, the internodal length usually reaches its maximum around the central nodal position and then gradually decreases toward both ends in a more or less symmetrical form. Thus, the distribution of internodal length can be expressed by the following equation,

$$y = \left( \frac{1}{\delta} \sqrt{2\pi} \exp - \frac{(X - \bar{X})^2}{2\delta^2} \right) \dots (5-1)$$

In equation (5-1), y indicates the length rate  $h_{(p)}$  of individual internodes, x the number of individual nodal position N,  $\bar{X}$  the average number of nodal position N calculated according to equation (5-2)  $\sigma$  the standard deviation of average nodal position calculated according equation (5-3).

$$\bar{N} = \frac{\sum_1^N Nh}{\sum_1^N h} \dots (6-2)$$

$$\sigma = \sqrt{\frac{\sum_1^N h(N - \bar{N})^2}{\sum_1^N h}} \dots (5-3)$$

$$\therefore h_{(p)} = \frac{1}{\delta \sqrt{2\pi}} \exp \frac{(N - \bar{N})^2}{2\delta^2} (5-4)$$

On the basis of equation (5-4), we select five culms of different size and calculate N,  $\sigma$  of distribution of their internodal length (Table 5-2).

Table 5-2 Distribution of internodal length

Locations	D.B. H. (cm)	Ht (cm)	Total $\Sigma$ N	Longest (N <sub>l</sub> )	Central (N <sub>c</sub> )	Ave. (N)	$\sigma \pm$	$\sigma/N$	A	B
Xiashui Xiangkeng	5.35	772	56	24	28	26.0	12.2	0.47	22.	0.9
	6.50	106	60	28	30	3	2	0.46	4	3
	8.00	0	56	28	28	29.3	13.5	0.46	28.	1.3
	11.1	109	66	27	33	0	5	0.49	2	0
	4	6	85	36	42	27.2	12.5	0.48	31.	1.4
	17.50	144				8	6		4	9
		6				30.3	14.9		43.	-
		244				2	3		1	1.9
		8				38.2	18.3		67.	0
						9	4		0	

Note: N,  $\sigma$ -parameters A- Average circumference of 1~5 internodes B- Average wall thickness of 1~5 internodes

As indicated in Table 5-2, the average node position (N) is located between the longest internode (N<sub>l</sub>) and the central node (N<sub>c</sub>) as generally expressed in  $N_l \leq N \leq N_c$ . For the practical purpose, N should be taken from 0~50 only. Thus, N of equation (4) can be calculated by

$$N = (N_l + N_c) / 2 \dots (5-5)$$

From Table 5-2, the deviation coefficient  $\sigma/N$  of five sampled culm varies between 0.46 and 0.49. Such a difference is so small that can be neglected. Thus, an assumed constant =0.5 is used in the practical calculation. The equation (5-4) can be written as

follows:

$$h_{(p)} = \left( \frac{1}{0.5N} \frac{1}{\sqrt{2\delta}} \right) \exp - \frac{1}{2} \left[ \frac{(N - \bar{N})}{(0.5N)} \right]^2$$

$$= \frac{1}{1.25N} \exp - 2 \left( \frac{N}{N} - 1 \right)^2 \dots (5-6)$$

Equation (6) is an expression of the internodal length rate of culms and can be used to calculate the actual length (h) of individual internodes by multiplying the internodal length rate by the total height (H) of the

culm. Consequently the distribution of internodal length of the culm also can be calculated accordingly.

$$h = h_{(p)} \times H = \left( \frac{H}{1.25N} \right) \text{EXP} \left[ -2 \left( \frac{N}{N} - 1 \right)^2 \right] dN \quad \text{-----(5-7)}$$

$$H_{(p)} = \int_1^N \left( \frac{H}{1.25N} \right) \text{EXP} \left[ -2 \left( \frac{N}{N} - 1 \right)^2 \right] dN \quad \text{-----(5-8)}$$

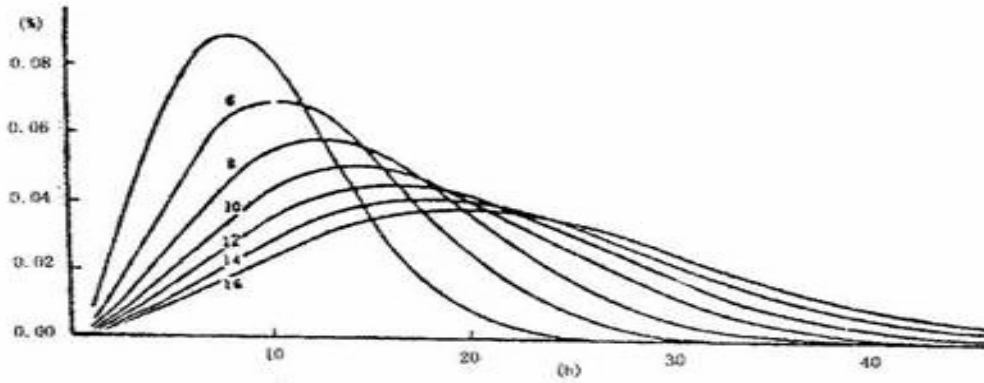


Fig.5-2 Distribution of internodal length rate

## 2. Distribution of internodal circumference

The internodes of bamboo culms are generally cylindrical. Their girths their circumference(L). The relation between internodal circumference and internodal diameter is  $L = \pi d$ . The internodal circumference decreases acrosspetally and is also related to the node position as expressed by

$$L = A \left( 1 - \frac{N}{2N} \right) \quad \text{-----(5-9)}$$

A is the circumference of the basal internodes and can be expressed by the average circumference of 1~5 internodes, i.e.  $A = 1/5 \sum L$ ; N the given node position; N the mean node position; either of them should be taken as a unit for the practical purpose.

## 3. Distribution of internodal wall-thickness

The wall-thickness of individual internodes decreases acrosspetally and is related to the node position as expressed by

$$Z = B \left( 1 - \frac{N}{2N} \right) \quad \text{-----(5-10)}$$

B is the wall-thickness of the basal internodes and can be expressed by the average wall-thickness of 1~5 internodes, i.e.

$$B = \frac{1}{5} \sum Z$$

## 4. Distribution of internodal volume

On the basis of length (h), circumference (L) and wall-thickness (Z) of an internode, its volume (V) can be calculated, i.e.  $V = h \cdot Z \cdot L$ . If  $L_1$  is the outer circumference of the internode,  $L_0$  the inner circumference,  $D_1$  the outer diameter, and  $D_0$  the inner diameter, then

$$L = \frac{1}{2} (L_1 + L_0)$$

$$; L_1 = \pi D_1;$$

$$L_0 = \pi D_0;$$

$$D_0 = D_1 - Z$$

$$L = \frac{0.5\pi}{(D_1 + D_0)} \quad \text{and} \quad L = 0.5\pi (D_1 + D_1 - Z) =$$

$$\pi D_1 - 0.5\pi Z$$

$$\text{Thus } V = hZ (L_1 - 0.5\pi Z) \quad \text{---- (5-11)}$$

If h from (5-7), L from (5-9) and Z from (5-10) are given to (5-11), we obtain

$$V = \left( \frac{H}{1.25N} \right) \text{EXP} \left[ -2 \left( \frac{N}{N} - 1 \right)^2 \right] \times$$

$$B \left( 1 - \frac{N}{2N} \right)$$

$$V = \left( \frac{H}{1.25N} \right) \text{Exp}^{-2} \left[ \frac{N}{N} - 1 \right]^2 \left[ AB \left( \left[ \frac{N}{N} - 1 \right] \right)^2 \right]$$

$$\left( \frac{HB}{1.25N} \right) \left( 1 - \frac{N}{2N} \right)^2$$

$$= \left( \frac{HB}{1.25N} \right) \left( 1 - \frac{N}{2N} \right)^2 \left( A - 0.5\pi B^2 \right) \left( 1 - \frac{N}{2N} \right)^2$$

----- (5-12)

Equation(5-12) is the distribution of internodal volume of culms and equation(13) is the accumulative distribution of internodal volume of culms. The number 1,2,3,...N of (5-13) should be unit for practical calculation.

$$V = \int_1^N V dN =$$

$$\int_1^N \left( \frac{HB}{1.15N} \right) (A - 0.5B\pi) \left( 1 - \frac{N}{2N} \right)^2$$

$$\text{Exp}^{-2} \left( \frac{N}{N} - 1 \right)^2 dN;$$

$$= \frac{HB}{1.25N} (A - 0.5\pi) B \int_1^N \left( 1 - \frac{N}{2N} \right)^2 \text{Exp}^{-2}$$

$$\left( \left[ \frac{N}{N} - 1 \right] \right)^2 dN \text{ -----(5-13)}$$

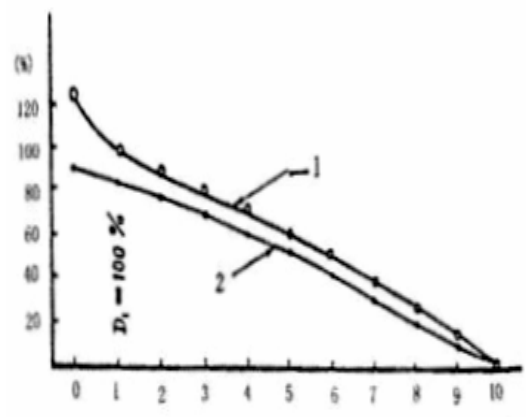


Fig.5-3 Relative culm form rate (1.Outerdiameter; 2.Inner diameter)

**Structural Model of Culm Form**

The culms of *Phyllostachys pubescens* display a general model of their structural form, though they vary greatly in diameter, height and wall-thickness.

**1.Variation of relative diameter of culms**

A culm can be divided into 10 parts which are called the relative height and can be numbered by 0,1,2,...9 from the base to the top. The ratio of a given diameter (D<sub>x</sub>) to that (D) at 1/10 height is the relative diameter (D%) as expressed by D% = D<sub>x</sub>/D<sub>0.1</sub> × 100%.

As indicated in Table 4-3 the relative diameter at a same relative height of culms is very similar regardless of their llocation, stand condition and size. In general, with 1/10 increase in relative height of culms upward, the relative diameter decreases about 10% (Fig.5-3).

In order to simulate the relationship between D% and H<sub>x</sub>, the theoretical value, standard deviation (σ) and correlation coefficient (r) can be calculated according to equation (5-14) and listed in Table 5-3.

$$D\% = 122 [ 1 - ( 0.1H_x )^3 ] - 18.5H_x ( 1 - 0.1H_x )$$

----- (5-14)

Table 5-3 Relative diameter of relative height of culms (D<sub>x</sub>/D<sub>i</sub>, %)

Locations	culms	Relative height(total height=10)									
		0	1	2	3	4	5	6	7	8	9
Jiangsi	407	124.16	100	90.79	81.65	72.14	62.15	51.76	40.41	28.56	16.22
Jiansu	148	129.10	100	91.50	82.28	72.22	61.84	50.34	38.98	27.43	15.01
Anhui	54	121.59	100	89.39	81.64	71.70	61.06	50.34	38.64	26.65	14.57
Zhejiang	15	127.46	100	89.07	80.75	70.13	59.80	50.75	40.67	28.69	16.64
Kuizhou	14	120.83	100	89.99	80.25	70.83	61.09	50.64	39.39	26.40	14.99
Av.or.Tot±σ	638	125.10	100	90.80	81.74	72.04	61.91	51.26	39.91	28.09	15.78
Cal.valus		2.4212	0	0.619	0.407	0.416	0.546	0.785	0.784	0.772	0.771

		2.00	105.2	1.40	9.90	9.80	0.50	1.20	1.30	9.90	6.40
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In polynomial simulation of  $D\%$  and  $H_x$ , the more nomials, the smaller standard deviation. But for general practice, a cubic curvilinear regression is enough. According to equation (5-14), a relative diameter and then an actual diameter at any relative height can be calculated.

The ratio of the breast-height diameter (D) to  $D_{0.1}$  with increase in diameter at breast height, and their relation is

$$(D/D_{0.1}) = 81.14 + 2.85D - 0.085D^2 \quad (5-15)$$

$$\text{From (5-15)} D_{0.1} = D (81.14 + 2.85D - 0.085D^2)^{-1} \quad (5-16)$$

then multiplying (5-14) by (5-16)

$$D_x = [122 [1 - (0.1H_x)^3] - 18.5H_x (1 - 0.1H_x)] / (81.14 + 2.85D - 0.085D^2) \quad (5-17)$$

If D and  $H_x$  are known,  $D_x$  can be calculated from (5-17).

## 2. Variation of wall-thickness rate of culms

The percentage of wall-thickness in outer diameter of culms, known as wall-thickness rate, is closely related to their size and parts. As seen in Fig.5-4, the wall-thickness rate decreases with increase in the breast-height diameter. In individual culms, the wall-thickness rate is always greater in their lower and upper parts than that in the middle.

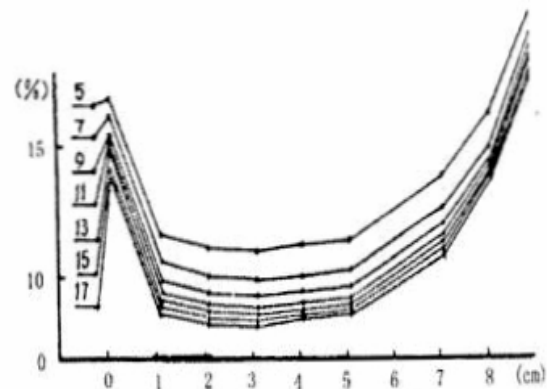


Fig.5-4. Wall-thickness rate (%) at relative height of culms (d.c. Diameter class Taimaoshan 1980)

The wall-thickness rate of culms varies considerably in different districts (Table 5-4). In general, the warm, humid climate promotes the height growth of culms, while the low temperature and low humidity favor the increase of the culm wall-thickness. In an area the bigger diameter of culms, the thicker culm wall-thickness. The relationship between breast-height diameter (D) and wall-thickness (Z) at breast height is:

$$Z = a + bD \quad (5-18)$$

According to equation (18), the following formulas can be derived. Taimaoshan, (121 culms):

$$Z = 0.07 + 0.092D \quad (5-19)$$

Table 4-4 Wall-thickness rate of relative height of culms (Z/D)

Relativ	0	1	2	3	4	5	6	7	8	9	Ave.
Yixing	17.77	11.88	10.92	10.73	10.30	9.83	11.09	13.03	14.44	21.36	13.14
Damaoshan	15.50	9.99	9.51	9.33	9.58	9.66	10.81	12.28	14.34	18.93	11.98
Chishui	13.60	9.34	8.99	8.83	8.94	8.81	9.59	10.45	12.74	14.83	10.61

Tienmnsan (82 culms):

$$Z = 0.25 + 0.086D \quad (5-20)$$

Funxing (109 culms):

$$Z = 0.27 + 0.084D \quad (5-21)$$

Yixing (55 culms):

$$Z = 0.33 + 0.086D \quad (5-22)$$

In the formulas mentioned above, b is relatively stable and can be estimated to be 0.086, while a varies considerably if different areas and is related to the annual mean temperature (t) and annual precipitation (R):

$$a = 0.72 - 1.85 R t 10^{-5} \quad (5-23)$$

Thus, the relationship between Z and D in different

districts can be obtained by substituting  $0.72 - 1.85 R t \times 10^{-5}$  and 0.086 for a, b in equation (5-19) respectively.

$$Z = 0.72 - 1.85 R t 10^{-5} + 0.086D \quad (5-24)$$

## 3. Variation of relative wall-thickness of culms

The ratio of a given wall-thickness ( $Z_x$ ) to the wall-thickness at 1/10 height ( $Z_{0.1}$ ) is known as relative wall-thickness ( $Z\%$ ) and can be expressed by

$$Z\% = (Z_x / Z_{0.1}) \times 100\%$$

As seen in Table 4-2, the relative wall-thickness at a relative height of culms is very similar regardless of their provenance, stand condition and size. On the basis of 251 culms from five districts, the relationship between

relative wall-thickness( $Z\%$ ) and relative height( $H_x$ ) of culms can be calculated by the least square fitting,

$$Z\% = 107.5 - 10.52H_x + 0.296H_x^2 - 0.118H_x^3 \quad (5-25)$$

According to (4-25), relative wall-thickness and actual wall-thickness at any relative height of a culm can be calculated if its wall-thickness at a given relative height is known. The relative wall-thickness at breast height  $[(Z/Z_{0.1}) \times 100\%]$  increases with increase in diameter at breast height. Their relationship is:

$$(Z/Z_{0.1}) \times 100\% = 2.14D + 81.44 \quad (5-26)$$

$$\therefore Z_{0.1} = (Z / (2.14D + 81.44)) \quad (5-27)$$

From (5-24):

$$Z_{0.1} = (0.72 - 1.85Rt10^{-5} + 0.086D) / (2.14D + 81.44) \quad (5-28)$$

$$\therefore Z\% = Z_x / Z_{0.1}$$

$$\therefore Z_x = Z / Z_{0.1} \quad (4-29)$$

Then  $Z\%$  from (5-25) and  $Z_{0.1}$  from (5-28),

$$Z_x = (0.72 - 1.85Rt10^{-5} + 0.086D) \times (107.5 - 10.52H_x + 0.296H_x^2 - 0.118H_x^3) / (2.14D + 81.44) \quad (5-30)$$

Consequently, the wall-thickness at different relative height of different culms from different areas can be calculated, if  $t$ ,  $R$  and  $D$  values are given. Structural Model of Fresh Weight of Culms

Table 5-5 Relative wall-thickness at relative height of culms ( $Z_x/Z_{0.1}$ )%

Locations	Relative height(total ht=10) of Culms									
	1.3m	0	2	3	4	5	6	7	8	9
Taimaoshan	131	188.0	85.138	76.237	68.226	61.375	53.365	46.014	37.233	26.202
Yuchishan	54	173.67	5.8485	4.6377.	6.1569	7.7960.	1.6153	3.9445	5.5134	6.8725
Xiasu	37	191.64	.4682.	5478.3	.7168.	2860.5	.5054.	.3647.9	.9939.	.1531.
Shimen	15	187.37	9585.3	775.96	9367.4	757.47	0151.8	143.41	3136.1	9723.4
Chishui	14	178.33	685.22	76.207	967.91	60.176	752.96	45.444	336.59	426.38
Ave.total	251	184.88	87.55	8.29	69.40	0.83	52.49	4.32	36.24	28.19
Theoretic value										

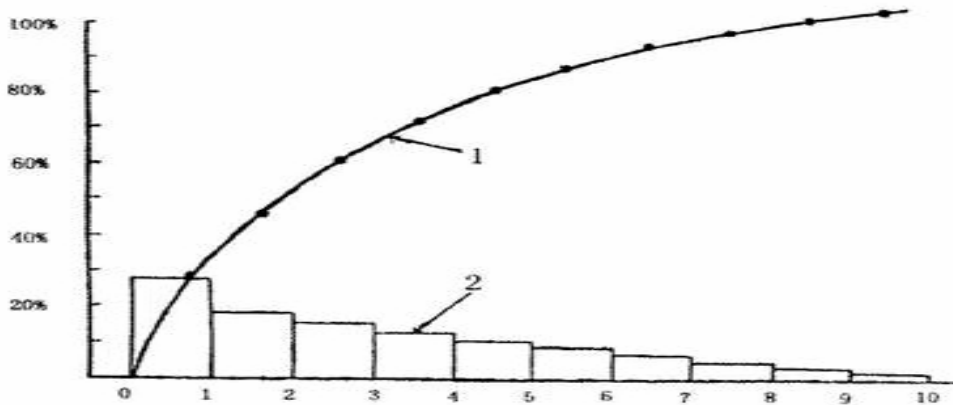


Fig.5-5 Weight percent of relative sections of culms  
 (1. Accumulative percent of section weight;2. Percent of section weight)

Table 5-6 Weight percent of relative section of culms

Locations	Section weight of culm													
	Culms				1	2	3	4	5	6	7	8	9	10
Jiangxi	24	28	14	1	26.	18.	14.	11.	9.4	7.4	5.3	3.5	1.8	0.7
Jiangsu					74	56	63	85	4	0	4	0	2	2
Zhejiang					26.	18.	15.	12.	9.5	7.2	5.0	2.9	1.4	0.5
Kui					88	94	08	23	8	6	5	8	8	2
Tot.or Ave.					27.	18.	13.	11.	9.5	7.2	5.2	3.6	2.0	0.8
					74	27	94	51	1	7	4	3	3	6
					26.	19.	15.	12.	9.3	7.1	5.2	3.0	1.5	0.3
					99	16	01	05	9	8	1	4	8	9
					27.	18.	14.	11.	9.4	7.2	5.2	3.2	1.7	0.6
					01	75	73	96	9	9	0	5	0	2